Sound field simulation method by combining finite difference time domain calculation and multi-channel reproduction technique

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Abstract: In order to auralize the room impulse responses obtained by the numerical analysis based on the wave theory, a multi-channel sound field simulation system using the Finite Difference Time Domain (FDTD) method has been developed. In this system, uni-directional impulse responses for four orthogonal directions in a 2-dimensional space are firstly calculated by the FDTD method and they are reproduced directly from four loudspeakers set in an anechoic room. At the center point of the reproduced sound field, we can hear the sound with 2-dimensional information assumed in the FDTD calculation. In this paper, the principle and the basic performance of this system are introduced and an example of subjective hearing test performed using this system is presented.

Keywords: Sound field simulation, Multi-channel reproduction, Auralization, FDTD Method

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1. INTRODUCTION

For the numerical analysis of acoustic problems, the Finite Difference Method, Finite Element Method and Boundary Element Method are being applied. Among them, the authors have been investigating the application of the Finite Difference Time Domain (FDTD) method [1–3] to sound field analysis [4,5]. As a significant merit of the FDTD method, transient acoustic phenomena can be directly calculated with relatively small memory size in a computer. By taking this advantage, the authors applied this method to analysis and visualization of sound propagation over barriers and in rooms [6,7].

As a technique to reproduce the sound in 3-dimensional sound fields, a very simple method has been contrived in our laboratory [8]. In this technique, six channel signals recorded through six uni-directional microphones orthogonally set on the rectangular coordinates are reproduced through six loudspeakers set in an anechoic room. Though this system is very simple in principle, it can accurately reproduce the sound with the original 3-dimensional information. In addition, this method has an advantage that the constraint on the listening condition is relatively loose as compared to the trans-aural reproduction technique and the listener can turn his/her head at the listening position.

By combining these ideas of numerical analysis and sound reproduction, the authors have constructed a new auralization technique [9]. In this paper, the principle and the basic performance of this technique are firstly introduced and a basic subjective hearing test on the effect of sound diffusion treatment in rooms performed by applying this technique is presented. This technique can be expanded to 3-dimensional sound field reproduction in principle but the calculation and the sound field reproduction mentioned in this paper are restricted to 2-dimensional sound field because of the limitation of the performance of the present computational system.

2. 4-CHANNEL SOUND FIELD SIMULATION SYSTEM

2.1. Outline of the System

Figure 1 shows the 4-channel sound field simulation system combining the 2-dimensional numerical analysis using the FDTD calculation and 4-channel sound reproduction system. In this system, the directional impulse responses in the directions of every 90 degrees at a receiving point in 2-dimensional sound field are firstly calculated by the FDTD method. Next, the calculated impulse response signals are reproduced directly through the four loudspeakers arranged at every 90 degrees in an anechoic room. Consequently, the sound with 2-dimen-
sional information assumed in the FDTD calculation can be realized and auralized at the center point of the sound reproduction system.

### 2.2. Calculation of the Directional Impulse Responses by the FDTD Method

In a 2-dimensional field, sound wave is expressed by the following partial differential equations. Equations (1) and (2) are the momentum equations in \(x\)- and \(y\)-directions, respectively, and the Eq. (3) is the continuity equation.

\[
\frac{\partial p(x,y,t)}{\partial x} + \rho \frac{\partial u_x(x,y,t)}{\partial t} = 0 \quad (1)
\]

\[
\frac{\partial p(x,y,t)}{\partial y} + \rho \frac{\partial u_y(x,y,t)}{\partial t} = 0 \quad (2)
\]

\[
\frac{\partial p(x,y,t)}{\partial t} + \kappa \left( \frac{\partial u_x(x,y,t)}{\partial x} + \frac{\partial u_y(x,y,t)}{\partial y} \right) = 0 \quad (3)
\]

where \(p\) is the sound pressure, \(u_x\) and \(u_y\) are the particle velocities in \(x\)- and \(y\)-directions, respectively, \(\rho\) is the density of the air and \(\kappa\) is the volume elastic modulus of the air.

By applying the staggered grid system with square-grids (\(\Delta x = \Delta y = \Delta h\)) shown in Fig. 2 to the sound field, the differential equations are transformed to the following discrete forms.

\[
u_x^{n+1}(i+1/2,j) = u_x^n(i+1/2,j) - \frac{\Delta t}{\rho \Delta h} \left( p^{n+1/2}(i+1,j) - p^{n+1/2}(i,j) \right) \quad (4)
\]

\[
u_y^{n+1}(i,j+1/2) = u_y^n(i,j+1/2) - \frac{\Delta t}{\rho \Delta h} \left( p^{n+1/2}(i,j+1) - p^{n+1/2}(i,j) \right) \quad (5)
\]

\[
p^{n+1/2}(i,j) = p^{n-1/2}(i,j) - \Delta t \left\{ \frac{u_x^n(i+1/2,j) - u_x^n(i-1/2,j)}{\Delta h} + \frac{u_y^n(i,j+1/2) - u_y^n(i,j-1/2)}{\Delta h} \right\} \quad (6)
\]

where \(\Delta h\) is the size of the square-grid and indices \(n, n+1/2, n-1/2\) and \(n+1\) denote time steps.

As the initial condition assuming an impulsive source, a smoothly continuous distribution of sound pressure described by the following equation was set (see Fig. 3).

\[
p(r) = \begin{cases} 
1 + \cos \pi \frac{r}{N \Delta h} & (r \leq N \Delta h) \\
0 & (r > N \Delta h)
\end{cases} \quad (7)
\]

where \(r\) is the distance of a grid point from the source position and \(N\) is the parameter indicating the spatial extent of the impulse. In order to make the initial condition smooth, \(N\) has to be more than 10 and \(N = 12\) was assumed in this study.

Under such an initial condition, the sound pressure and the particle velocities at each grid point are calculated successively based on Eqs. (4), (5) and (6).
In order to calculate the directional impulse responses for four orthogonal directions, an arbitrary directivity factor is assumed and set in every 90 degrees by rotating its direction at the receiving point. The direction of each directivity factor is expressed using a direction vector \( d = (d_x, d_y) \), where \(|d| = 1\). In this study, as the directivity factor, cardioid function expressed by the following equation was chosen and each direction vector was set parallel to \( x \)- or \( y \)-axis (see Fig. 4).

\[
f(\theta) = \frac{1 + \cos \theta}{2}
\]

where \( \theta \) is the angle of the incident sound to each directivity factor at the receiving point.

By multiplying the instantaneous sound pressure at the receiving point and the directivity factor, each of the directional impulse responses at the receiving point is calculated.

\[
p_{\text{directional}}^{n+1/2}(i, j) = p_{\text{directional}}^{n+1/2}(i, j) \cdot f(\theta)
\]

In the calculation of the angle \( \theta \), the arrival direction of the sound at the receiving point is obtained by calculating the instantaneous sound intensity. In the staggered grid system shown in Fig. 2, the sound intensities in \( x \)- and \( y \)-directions at the grid point \((i, j)\) are approximately calculated as the product of the sound pressure and the particle velocities in \( x \)- and \( y \)-directions, respectively. Here, the particle velocities in \( x \)- and \( y \)-directions at the grid point \((i, j)\) can be calculated as the spatial averages of the particle velocities at the points of \((i-1/2, j)\) and \((i+1/2, j)\) and those at the points of \((i, j-1/2)\) and \((i, j+1/2)\), respectively, as follows.

\[
I_x^{n+1/2}(i, j) = p_{\text{directional}}^{n+1/2}(i, j) \cdot \frac{u_x^n(i + 1/2, j) + u_x^n(i - 1/2, j)}{2}
\]

\[
I_y^{n+1/2}(i, j) = p_{\text{directional}}^{n+1/2}(i, j) \cdot \frac{u_y^n(i, j + 1/2) + u_y^n(i, j - 1/2)}{2}
\]

Thus, the angle of \( \theta \) for each directivity factor is calculated using the direction vector as follow.
3. REPRODUCTION ACCURACY

In order to examine the reproduction accuracy of the simulation system, the following basic investigations were performed on a free field and three types of 2-dimensional rooms. In the calculation in these studies, the spatial grid size of 0.01 m and time-step of 0.02 ms were set and the spectrum characteristic of the impulse sound source is as shown in Fig. 3(b).

The 4-channel reproduction system was constructed in an anechoic room of 7 meters cubed, in which four loudspeakers (TANNOY, T12) were set crossing at right angles on an arc of 2 m radius as shown in Fig. 5(b). In order to correct the differences in reproduced sound pressure and frequency characteristics between the four loudspeakers, four spectrum-equalizers (SONY, SRPE210) were used, so that the differences were retained within $\pm 1.0$ dB in each octave band from 63 to 4kHz.

3.1. Study on the Arrival Direction of Sound in a Free Field

In order to simulate a free field condition in the FDTD calculation, the concept of the “perfectly matched layer” (PML) which was initially developed to solve the Maxwell equation [10] was applied and the hypothetical non-reflection boundary layer was set around the sound field under consideration as shown in Fig. 5(a) [11]. In order that the incident sound is attenuated in the layer, the hypothetical flow resistance $R$ is assumed in the momentum equations as follows.

\[
\theta = \cos^{-1}\left\{ \frac{-d_x \cdot I_x^{n+1/2}(i, j) - d_y \cdot I_y^{n+1/2}(i, j)}{\sqrt{(I_x^{n+1/2}(i, j))^2 + (I_y^{n+1/2}(i, j))^2}} \right\} (12)
\]

According to the concept of PML, the characteristic impedance of the medium in the layer is assumed to be equal to that in the sound field under consideration and the hypothetical attenuation factor $Q$ is assumed in the continuity equation as follows. This is just for convenience in numerical analysis and not realized physically.

\[
\frac{\partial p(x, y, t)}{\partial t} + \rho \left( \frac{\partial u_x(x, y, t)}{\partial x} + \frac{\partial u_y(x, y, t)}{\partial y} \right) + Q \cdot p(x, y, t) = 0 (15)
\]

When assuming that the characteristic impedance in the layer is $\rho c$, $Q$ is expressed using $R$ as follows.

\[
Q = \frac{R}{\rho} (16)
\]

In this study, it is assumed that $R$ gradually increases with the increase of the distance from the border between the sound field and the hypothetical layer and $R$ is expressed as follows.

\[
R = R_{\text{max}} \left( \frac{l}{W} \right)^m (17)
\]

where $R_{\text{max}}$ is the maximum value of $R$ at the end of the hypothetical layer, $W$ is the width of the layer and $l$ is the distance from the border between the sound field and the hypothetical layer (see Fig. 6). Here, in order to eliminate the reflection at the end of the hypothetical layer, the value of $R$ is assumed to be equal to $R_{\text{max}}$ for the sound wave reflected at the end of the hypothetical layer just for
convenience in this case, too. In this study, $R_{\text{max}} = 25 \text{ Ns/m}^3$, $W = 0.8 \text{ m}$ and $m = 2$ were chosen from the result of the following trial-and-error study. A 2-dimensional sound field of 8 meters square was assumed and hypothetical non-reflection boundary layer was set around the field. An impulse was emitted from a source point set at the center point of the sound field and the reflections from the boundaries at two points in the field was examined. By changing the values of the parameters $R_{\text{max}}$, $W$ and $m$ widely, their optimum values were decided so that the energy of the reflections from the boundaries became minimum.

In the study on a free field, the position of the sound source and the receiving point were set as shown in Fig. 5(a) and the directional impulse responses at the receiving point were calculated for each sound source position ($S_0$ to $S_{90}$).

1) Sound intensity measurement

The 4-channel directional impulse responses calculated by the FDTD method for each sound source were convolved with a pink noise. These synthesized signals were reproduced through the 4-channel loudspeaker system set in an anechoic room and the sound intensities in x- and y-directions in the horizontal plane at the center point in the reproduction field were measured separately. In the measurement, a pair of 1/2 in. condenser microphones (B&K 4181) was used. The separation distance of the microphones was set to be 12 mm for the measurements between 250 Hz to 2k Hz octave bands and 50 mm for 125 Hz band.

From the measurement results, the sound intensity vectors were calculated in each octave band from 125 Hz to 2k Hz as shown in Fig. 7. In these figures, the length and the direction of each arrow indicate the relative sound intensity level and the arrival direction of the sound, respectively. In each result, it is seen that each arrow approximately points to the corresponding sound source position assumed in the FDTD calculation and the length of the sound intensity vector is almost uniform. To see the results in more detail, however, slight discrepancies are seen between the experimental result and the assumed angle. These simulation errors might be caused by the slight differences of system gain among the 4-channel reproduction system and imperceptible error in positioning of the intensity probe.

2) Subjective judgment of arrival direction of sound

Next, the arrival direction of the sound was examined by subjective hearing test. For this experiment, the 4-channel impulse responses calculated for every 30 degrees were convolved with an intermittent pink noise (1 s on-time and 0.5 s off-time, three bursts) and they were reproduced through the 4-channel loudspeaker system. The subject sitting at the listening position was asked to judge the direction in which the test sound arrived. In this test, the directions of every 30 degrees were numbered from 0 to 11 as shown in Fig. 5(b). During the test, the subject was allowed to turn his/her head to judge the arrival direction.

Seven graduate and undergraduate students with normal hearing ability participated in this experiment. Figure 8 shows a scene of the subjective test.

Figure 9 shows the test result, in which the diameter of each circle indicates the relative number of the judgment. In this result, the directions assumed in the FDTD calculation and those judged by the subjects are in very good agreement.

### 3.2. Impulse Response in 2-Dimensional Rooms

In order to examine the reproducibility of room impulse response, the following studies were performed. As the...
sound fields under consideration, three types of 2-dimensional rooms shown in Fig. 10 were chosen and the sound source position and the receiving position were set in each room. In the calculation, it was assumed that the normal acoustic impedance on the room boundaries consisted only of the real part and a constant sound absorption coefficient ($\alpha = 0.2$) was assumed for all boundaries.

The omni-directional and 4-channel directional impulse responses and the instantaneous sound intensity in $x$- and $y$-directions at the receiving point were calculated by the FDTD method.

(1) Echo-diagram

The calculated directional impulse response signals were reproduced through the 4-channel loudspeaker system and the omni-directional impulse response was measured at the center point in the reproduced sound field. Figure 11 shows the comparison of echo-diagram between the calculation and the measurement. These echo-diagrams were obtained by passing the omni-directional impulse response signal through a numerical RMS detector of 1 ms time constant. As seen in the figure, the calculated results and the measured ones are in very good agreement.

(2) Instantaneous sound intensity

The calculated directional impulse response signals were again reproduced and the instantaneous sound intensities at the center point in $x$- and $y$-directions were measured separately using the intensity probe (B&K 4181 with 12 mm spacing). In order to express the direction and magnitude of the propagating sound energy in transient state, the instantaneous sound intensity was averaged for every 0.5 ms time interval and the sound intensity vector for each interval was calculated. Figure 12 shows the comparison of sound intensity vector between the calculation and the measurement in the form of radar-charts. Here the magnitude of the sound intensity is expressed in relative level in the range of 20 dB.

In these figures, it is seen that the intensity vectors in the calculation and the measurement are in fairly good agreement for all of the three cases as a whole. To see the results in more detail, however, some discrepancies are
seen between the calculated results and the measured ones. That is, the arrival direction of the directional sound is well reproduced, whereas slight discrepancies are seen in the direction and the magnitude of the early reflections and the number of the vectors observed in the radar-charts is larger in the measurement than in the calculation. These differences might be attributed mainly to the transient effect of the loudspeakers used in the reproduction system. That is, the duration time of the impulse reproduced through a loudspeaker inevitably becomes longer than that of the ideal impulse. By the effect of time-averaging of the instantaneous sound intensity mentioned above, the number of the vectors in the measurement becomes larger than that in the calculation. In addition, the reciprocating reflections between the opposite loudspeakers and the slight error of positioning of the intensity probe might affect the simulation accuracy.

4. SUBJECTIVE EXPERIMENT ON THE EFFECT OF SOUND DIFFUSERS

As an application of the 4-ch. system to room acoustics, a subjective experiment on the effect of fluttering echo prevention by sound diffusers was performed. Figure 13 shows the rectangular room under investigation and two types of sound diffusers shown in Fig. 14 (a: Triangular and b: Column) were attached to the room boundaries. For each type of diffuser, four variations in dimension shown in Table 1 were assumed. As the boundary condition, it was assumed that the normal acoustic impedance on the room boundaries consisted only of the real part and a constant

Fig. 11 Comparisons of echo-diagram between the calculation and measurement.

Fig. 12 Comparisons of instantaneous sound intensity between the calculation and measurement.
sound absorption coefficient was assumed for all boundaries. In the case of the bare room condition (without diffusers), the absorption coefficient of 0.2 was set. In the cases where the diffusers were attached, the sound absorption coefficient was assumed so that the equivalent sound absorption area in 3-dimensional rooms became equal to that under the bare room condition. The value for each sound absorption length, which corresponds to equivalent absorption coefficient was assumed so that the equivalent cases where the diffusers were attached, the sound absorption coefficient of 0.2 was set. In the 2-dimensional rectangular room under investigation.

In the subjective experiment, the impulse responses were presented to the subject through the 4-channel loudspeaker system in random order. After hearing each impulse response, the subject judged the strength of fluttering echo in 5-step categories shown in Table 2. Five subjects participated in this experiment. In order to check the repeatability of the judgment, the test was repeated four times for each subject. As a result, relatively high correlation was obtained both in the result of the individual test-retest check and mutual correlation among all of the subjects. The average of the correlation coefficient was 0.88 for the former and 0.92 for the latter, respectively. From this result, the judgment results by all of the subjects were averaged.

For the subjective hearing test, 4-channel impulse responses at each receiving point were calculated under the conditions of with and without the diffusers. In the FDTD calculation, the spatial grid size of 0.01 m and the time-step of 0.02 ms were assumed in this case, too.

In the sound field simulation in room acoustics, it is earnestly desired that the calculated results by numerical analysis can be auralized for subjective judgment. For this aim, a new simulation method has been developed by combining the numerical analysis based on FDTD calculation and multi-channel reproduction technique. Its accuracy of sound field reproduction has been confirmed through acoustic measurements and subjective judgment test. As an application of this method to room acoustic

Table 1 Dimensions of the sound diffusers and absorption coefficient of boundaries under investigation.

<table>
<thead>
<tr>
<th>Type</th>
<th>Sizes (unit: m)</th>
<th>Absorption coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular-1</td>
<td>$W = 0.4 \ H = 0.06$</td>
<td>$\alpha = 0.19$</td>
</tr>
<tr>
<td>Triangular-2</td>
<td>$W = 0.75 \ H = 0.11$</td>
<td>$\alpha = 0.19$</td>
</tr>
<tr>
<td>Triangular-3</td>
<td>$W = 1.5 \ H = 0.23$</td>
<td>$\alpha = 0.19$</td>
</tr>
<tr>
<td>Triangular-4</td>
<td>$W = 3.0 \ H = 0.45$</td>
<td>$\alpha = 0.19$</td>
</tr>
<tr>
<td>Column-1</td>
<td>$W = 0.4 \ D = 0.4$</td>
<td>$\alpha = 0.13$</td>
</tr>
<tr>
<td>Column-2</td>
<td>$W = 0.75 \ D = 0.4$</td>
<td>$\alpha = 0.08$</td>
</tr>
<tr>
<td>Column-3</td>
<td>$W = 1.5 \ D = 0.4$</td>
<td>$\alpha = 0.11$</td>
</tr>
<tr>
<td>Column-4</td>
<td>$W = 3.0 \ D = 0.4$</td>
<td>$\alpha = 0.15$</td>
</tr>
</tbody>
</table>

\[ l_D \alpha_D = l_B \cdot \alpha_B \]  

(18)

where $l_B$ (94.56 m) is the boundary length and $\alpha_B$ (0.2) is the absorption coefficient set for the bare room condition, and $l_D$ is the boundary length and $\alpha_D$ is the sound absorption coefficient for each room condition with diffusers.

For the subjective hearing test, 4-channel impulse responses at each receiving point were calculated under the conditions of with and without the diffusers. In the FDTD calculation, the spatial grid size of 0.01 m and the time-step of 0.02 ms were assumed in this case, too.

In the subjective experiment, the impulse responses were presented to the subject through the 4-channel loudspeaker system in random order. After hearing each impulse response, the subject judged the strength of fluttering echo in 5-step categories shown in Table 2. Five subjects participated in this experiment. In order to check the repeatability of the judgment, the test was repeated four times for each subject. As a result, relatively high correlation was obtained both in the result of the individual test-retest check and mutual correlation among all of the subjects. The average of the correlation coefficient was 0.88 for the former and 0.92 for the latter, respectively. From this result, the judgment results by all of the subjects were averaged.

Figures 15(a) and (b) show the results for the triangular and the column diffusers, respectively. In these figures, each plot means the arithmetic average of the category number answered by all of the subjects for each diffuser condition at each receiving point. In the cases of the triangular diffusers, it is seen that the effect of fluttering echo prevention increases with the increase of the size of the diffusers. On the other hand, in the cases of the column diffusers, the effect is the highest when the column interval is 1.5 m. It indicates that an optimum scale exists for the column interval to prevent fluttering echo.

From the results of this experiment, it has been confirmed that the effect of fluttering echo prevention by sound diffusers is much influenced by their shapes and dimensions.

5. CONCLUSIONS

In the sound field simulation in room acoustics, it is earnestly desired that the calculated results by numerical analysis can be auralized for subjective judgment. For this aim, a new simulation method has been developed by combining the numerical analysis based on FDTD calculation and multi-channel reproduction technique. Its accuracy of sound field reproduction has been confirmed through acoustic measurements and subjective judgment test. As an application of this method to room acoustic
study, the effectiveness of sound diffusion treatments on room surfaces was investigated by assuming a basic room shape and two types of diffusers. As a result, it has been found that the simulation method is applicable to the check of proper shape and size of sound diffusers.

The numerical analysis and sound field reproduction performed in this study were restricted to 2-dimensional sound fields because of the limitation of calculation performance of the computer used in this study. In the near future, we will expand the method to 3-dimensional sound field simulation and make it useful auralization technique of room acoustics with the progress of computational technology.

REFERENCES


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